

# Chapter 1

## 1. Introduction

### 1.1 The Fundamental Building Blocks

Currently, the Standard Model is the most widely accepted and experimentally tested theory that describes the composition of matter and how it interacts. There are two major categories of particles that make up the building blocks of the universe: fermions and bosons. The fundamental fermions are split into two families: quarks and leptons. These particles come together to form all of the matter in the universe. Particles such as protons and neutrons are made of quarks, and the electron is an example of a lepton. Bosons are the mediating particles in the interactions in matter. The photon is an example of a boson.

The distinguishing property that separates the fermions from the bosons is spin. All of the fermions have half-integer spin and obey Fermi-Dirac statistics while all of the bosons have integer spins and obey Bose-Einstein statistics.

When matter interacts, it does so by the exchange of a boson called a gauge boson. At sufficiently low exchange energies there are four types of interactions, also known as the four forces. They are the Strong Force, Electromagnetic Force, Weak Force, and Gravity. Each force is mediated by its own type of gauge boson. Whether or not a particle interacts via one of the four forces depend on its properties. If a particle lacks a certain property, it simply cannot interact with other matter via that force. Inversely, if a particle possesses a certain property, it can and will interact with other matter that also possess this property. The Strong and Weak Forces are known as the nuclear forces and are limited in range. Both have a range of the size of a proton or less. The strong force is what is responsible for binding the protons and neutrons together in the nucleus. It is aptly named as it is the strongest of the four forces and is always attractive. The weak force is responsible for many types of particle decay, including the expected decay modes of the top quark. Its name can be misleading, as it is not the weakest of the four forces; rather, it is just the weakest of the two nuclear forces. The electromagnetic force and gravity are both infinite in range. Outside of the nucleus of an atom, these are the only two forces that are important. The electromagnetic force is responsible for binding electrons to the nucleus of an atom. It is attractive when two particles have opposite signs and repulsive when

they are the same. Also, the electromagnetic force is proportional to the size of the charged objects, and inversely proportional to the square of the distance between them. It is about two orders of magnitude weaker than the strong force but about four orders of magnitude stronger than the weak force\*. Gravity is most familiar of the forces, but by far the weakest. It is about thirty-seven orders of magnitude weaker than the electromagnetic force. It only appears to be strong in everyday life due to the size of astrological bodies, such as the Earth. It is always attractive, proportional to the masses of the two bodies in question, and inversely proportional to the square of the distance between them. Due to its relative weakness, it is not significant in high energy particle interaction and can be ignored.

Force	Boson	Symbol	Interactive Property
Strong	Gluon	$g$	color
Electromagnetic	Photon	$\gamma$	electric charge
Weak	W and Z	$W^{\pm} Z^0$	weak charge
Gravity	Graviton*	$G$	mass

The distinguishing property that separates the fundamental fermions into the two families is color. The quarks all possess color and the leptons do not. This means that the only fundamental fermions that interact strongly are the quarks. Gluons also possess color and will interact with both the quarks and each other via the strong force. They are the only bosons that have this property. The remaining properties are spread through both families of fermions and the bosons. Each family has six members, which are separated into three generations each containing two particles.

Generation	I	II	III
electric charge	Quarks		
$+\frac{2}{3}e$	Up	Charm	Top
$-\frac{1}{3}e$	Down	Strange	Bottom
electric charge	Leptons		
$-e$	Electron	Muon	Tau
0	Electron Neutrino	Muon Neutrino	Tau Neutrino

For each particle there exists an anti-matter particle. They have the same masses, but the signs of their quantum number will be switched. For example, the proton( $p$ ) and anti-proton( $\bar{p}$ ) both have a mass of 938.3MeV and a spin of  $\frac{1}{2}\hbar$  but the charge of the proton is  $+e$  and the

charge of the anti-proton is  $-e$ . When a matter particle comes in contact with its anti-matter particle, they will annihilate and produce a boson.

## 1.2 Leptons

There are six leptons which easily break down into two groups: charged leptons and neutrinos. All leptons have half-integer spin ( $\frac{1}{2}\hbar$ ). The charged leptons all have an electric charge of  $-e$  ( $e \approx 0.303(\hbar c)^{\frac{1}{2}}$ ) and the neutrinos are all neutral. This means that the charged leptons are all affected by the electromagnetic force and the neutrinos are not. The charged leptons also have significant masses, the lightest being the electron and the heaviest being the tau, with the muon falling in the middle. While the electron is stable, the tau and muon both have a finite lifetime and will decay. The neutrinos have been shown to have mass from neutrino flavor changing, but their masses are extremely small ( $m < 2eV$ ), and for the purposes of this dissertation will be treated as massless. All six of the leptons have a property called weak charge which couples them to the weak force and, as stated above, none of them possess color so they do not couple to the strong force. So, the charged leptons all couple to three of the four forces while, if we neglect their tiny masses, the neutrinos only couple to one of the forces. From an experimental point of view, this makes the neutrinos extremely difficult to detect and in the case of CDF, undetectable. There are three generations of leptons, each containing one charged lepton and its associated neutrino. The first generation contains the electron( $e^-$ ) and electron neutrino( $\nu_e$ ), the second contains the muon( $\mu^-$ ) and the muon neutrino( $\nu_\mu$ ), and finally the third contains the tau( $\tau^-$ ) and the tau neutrino( $\nu_\tau$ ). Each of the types of neutrinos is associated with their charged lepton through experimentally observed conservation laws.

Generation	I	II	III
Charge Leptons	$e^-$	$\mu^-$	$\tau^-$
mass	0.5109 MeV	105.7 MeV	1.776 GeV
lifetime	stable	2.2 $\mu s$	290.6 fs
charge	$-e$	$-e$	$-e$
Neutrinos	$\nu_e$	$\nu_\mu$	$\nu_\tau$
mass	$< 2eV$	$< 2eV$	$< 2eV$
lifetime			
charge	0	0	0

There are also six anti-leptons, the positron( $e^+$ ), the mu plus ( $\mu^+$ ), the tau plus ( $\tau^+$ ), and

the three anti-neutrinos ( $\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau$ ).

### 1.3 Quarks

There are six quarks that, like the leptons, can be separated into two groups by charge. Unlike the leptons, none of the quarks are neutral, nor do they have integer values of  $e$  charge. Instead, one group consists of quarks with a fractional charge  $+\frac{2}{3}e$  and the other with  $-\frac{1}{3}e$ . Since, just like the leptons, they are fermions, they all also have a spin of  $\frac{1}{2}\hbar$ . All of the quarks are massive and have weak charge. In addition, quarks carry a special property called color. Color, in this context, is not the same thing as the concept of color as it would relate to visible light. The property of color is, in essence, strong charge - the property that couples to the strong force. Unlike electric charge, it is neither positive nor negative. The strong charge is called color because it comes in three varieties that, when mixed, combine in the same way as the colors of visible light. The three varieties are red, green, and blue (RGB). There are also six anti-quarks which possess anti-color ( $\bar{R}\bar{G}\bar{B}$ ). The six 'flavors' of quarks are Up(u), Down(d), Strange(s), Charm(c), Bottom(b), and Top(t), each of which has an anti-matter partner ( $\bar{u}, \bar{d}, \bar{s}, \bar{c}, \bar{b}, \text{ and } \bar{t}$ ).

Together, these twelve particles combine in different ways to form a family of composite particles called Hadrons. Hadrons can be identified by their valence quark - the combination of quarks of which they are composed. Hadrons come in two groups; baryons and mesons. Baryons are composed of three quarks or three anti-quarks and mesons are composed of a quark and anti-quark pair. Even though all hadrons are made of quarks, which carry color, they are all colorless. The colors of the quarks in a hadron must add up in a way that is colorless. This accounts for the fact that all observed hadrons are colorless and that a single quark has never been observed in nature. For example, the proton is a baryon, which has three valence quarks, each of which must carry a different color. Just like when adding red, green, and blue light gives white light, a red, green, and blue set of quarks will result in a hadron that is colorless. Another example is the  $\pi^+$  meson, which has a d quark and  $\bar{d}$  pair of valence quarks. In this case, the color of the quark must be the opposite to the anti-color of the anti-quark: red and anti-red or blue and anti-blue.

Since baryons are made up of three quarks, the total spin of the three quarks will add up

to something that is a half-integer spin. Therefore baryons are fermions. Two examples are the proton and the neutron. The proton is made up of two u quarks and a d quark and the neutron is made up of one u quark and two d quarks. Also, since mesons have two quarks, the total spin will add up to something that has an integer spin, and therefore mesons are bosons. Both the proton and the  $\pi^+$  meson have the same charge, but are made up of different quarks. The neutron is neutral, but has two of the same quarks as the proton. This is why the charges of the quarks must be fractional.

Similar to the leptons, not all quarks are stable. The u and d quarks are the only stable quarks, the s, c, b, and t quarks all decay quickly. This is why all the stable hadrons are made of u and d quarks, while hadrons that are made with one or more of the other quarks must be made in the lab.

Generation	I	II	III
	$u$	$c$	$t$
mass	1.5 to 3.3 MeV	1.27 GeV	
charge	$+\frac{2}{3}e$	$+\frac{2}{3}e$	$+\frac{2}{3}e$
	$d$	$s$	$b$
mass	3.5 to 6.0 MeV	104 MeV	4.2 GeV
charge	$-\frac{1}{3}e$	$-\frac{1}{3}e$	$-\frac{1}{3}e$

## 1.4 Gauge Bosons

There are four types of gauge bosons, one for each of the four types of interactions. These bosons differ from mesons in that they are not composite particles made from the bound state of a quark and anti-quark. Aside from the fact that they have an integer spin, they differ, as elementary particles, from the fundamental fermions in that they are not matter, but instead quanta of their associated fields. Though similar to the fundamental fermions, some of the gauge bosons do possess some of the same properties.

The gauge boson associated with the strong force is called the gluon( $g$ ). It is a massless, spin one particle. It couples to the strong force through color, and even possesses color itself. This means that gluons can interact with other gluons strongly, which is what is responsible for the short range of the strong force. The way in which gluons possess color is not as simple as the quarks. There are not simple red, green, and blue gluons. There are eight independent color

states of gluons that are represented by linear combinations of pairs of color and anti-color. For example, one such state could be  $r\bar{b} + b\bar{r}$ .

The photon( $\gamma$ ) is probably the most commonly known gauge boson and is associated with the electromagnetic force. Photons are massless, colorless, and neutral spin one particles. They do not interact with each other. They can be characterized by the energy that they carry, which is related to their frequency. They couple electromagnetically with charged particles.

The weak force has three gauge bosons associated with it. They are the  $Z^0$ ,  $W^+$ , and  $W^-$ . They are all massive,  $M_z \approx 91\text{GeV}$  and  $M_w \approx 80\text{GeV}$ , which is why the weak force has such a short range. Its range is considerably shorter than even the range of the strong force. The  $W^+$  and  $W^-$  both carry an electric charge of  $+e$  and  $-e$  respectively. This gives them the ability to interact with each other and other charged particles electromagnetically. Otherwise, they are responsible for the weak force and interact with particles that possess weak charge.

The fourth type of gauge boson is the Graviton. The Graviton has yet to be observed and is still theoretical. According to theory, the graviton is massless and has a spin of two. It gives rise to the force called Gravity, and couples to anything with energy and momentum - which means it couples to everything. According to some higher dimensional theories, [citation] the graviton may have the special ability to travel into other dimensions. They use this to help explain why gravity appears to be so much weaker than the other three forces.

Force	Strong	Electromagnetic	Weak	Gravity
Boson	Gluon	Photon	IVB	Graviton
Symbol	g	$\gamma$	$W^\pm, Z^0$	G
Spin	1	1	1	2
Mass	0	0	80 GeV, 91 GeV	0
Charge	0	0	$\pm e, 0$	0
Coupling	Color	Electric Charge	Weak Charge	Energy and Momentum

## 1.5 The Standard Model

The Standard Model is a local gauge field theory that describes how all of the known and a few of the theoretical particles interact. It is very well supported by experimental data but is incomplete. First of all, it does not include gravity, mainly due to the fact that a quantized field theory of gravity has not yet been discovered. Second, the Standard Model contains

twenty-three arbitrary parameters which include the twelve fermion masses, three force coupling strengths, two Higgs parameters, four CKM parameters, and a QCD parameter.

In Physics, both in Classical and Field Theory, it's important to be able to describe the motions of things. One of the most powerful formalisms for this is the Euler-Lagrange Equation. It was originally formalized classically and was eventually generalized to work with Relativity and Field Theory.

$$\partial_\mu \left( \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \right) = \frac{\partial \mathcal{L}}{\partial \phi_i} \quad (1)$$

At the heart of this equation is the Lagrangian,  $\mathcal{L}$ . Once the proper Lagrangian can be written down for a system, everything that needs to be known about that system can be derived from it. In the case of Particle Physics, the Lagrangian tells the story of how particles and fields interact with each other. An example is the Dirac Lagrangian.

$$\mathcal{L} = i\hbar c \bar{\psi} \gamma^\mu \partial_\mu \psi - mc^2 \bar{\psi} \psi \quad (2)$$

$\psi$  describe a spinor field, particle of spin  $\frac{1}{2}$ , with an adjoint field  $\bar{\psi}$  and mass  $m$ . This Lagrangian describes a free particle and consists of a kinetic term and a mass term.

As stated above, the Standard Model is a local gauge field theory, which means that it is invariant under local gauge transformations.  $\psi$  has both a real and imaginary part, however  $\bar{\psi}\psi$  is completely real and describes things that can be observed in reality. If a transformation is applied to  $\psi$  such that  $\psi \rightarrow e^{i\theta}\psi$ , where  $\theta$  is a real number and describes how much of  $\psi$  is real and how much is imaginary, the Lagrangian will be unchanged. This type of transformation is called a Global Gauge Transformation, and if the Lagrangian remains unchanged, it exhibits global gauge invariance. However, if  $\theta$  is a function of position,  $\theta(x)$ , then the transformation  $\psi \rightarrow e^{i\theta(x)}\psi$  is a Local Gauge Transformation. Eqn.2 is not invariant under local gauge transformations, however, if an additional term is added,  $-(q\bar{\psi}\gamma^\mu\psi)A_\mu$ , the Lagrangian becomes

$$\mathcal{L} = i\hbar c \bar{\psi} \gamma^\mu \partial_\mu \psi - mc^2 \bar{\psi} \psi - (q\bar{\psi}\gamma^\mu\psi)A_\mu \quad (3)$$

and local gauge invariance is achieved.  $A_\mu$  describes a new field, called a gauge field, which transforms like ,  $A_\mu \rightarrow A_\mu - \partial_\mu \lambda(x)$ .  $\lambda(x) \equiv -\frac{\hbar c}{q}\theta(x)$  and  $q$  is the charge of the particles. The new term in the Lagrangian describes how a spinor,  $\psi$ , interacts with a gauge field,  $A_\mu$ . To

make the Lagrangian complete, "free" terms have to be added for the gauge field. The Proca Lagrangian describes a spin 1 vector field,

$$\mathcal{L} = -\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu} + \frac{1}{8\pi}\left(\frac{m_A c}{\hbar}\right)A^\nu A_\nu \quad (4)$$

where  $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$  and  $m_A$  is the mass of the vector field. The first term in this Lagrangian works out just fine, but the second term has a problem: it is not locally gauge invariant. However, if the mass of the gauge field ( $A_\mu$ ) is zero, then the second term will vanish and the Lagrangian will be

$$\mathcal{L} = i\hbar c \bar{\psi} \gamma^\mu \partial_\mu \psi - mc^2 \bar{\psi} \psi - (q \bar{\psi} \gamma^\mu \psi) A_\mu - \frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu}. \quad (5)$$

It is obvious now that  $A_\mu$  is the electromagnetic potential and the Lagrangian describes how Dirac fields interact with the electromagnetic field. Or, in particle terms, how fermions like electrons interact with the photon.

The gauge transformation described above can be thought of as a  $1 \times 1$  matrix, and the fields  $\psi$  and  $A_\mu$  as single component vectors. These matrices are unitary; the product of their hermitian adjoint with themselves is one. The group of all such unitary matrices is called  $U(1)$ , and the symmetry is called  $U(1)$  gauge invariance. In a similar manner, Quantum Chromodynamics gives rise to a very similar Lagrangian; however it has some important differences. Since each flavor of quark can come in three different colors,  $\psi$  will now have three components, one for each color. Also, the local gauge transformation can be represented by a  $3 \times 3$  matrix which belongs to the  $SU(3)$  group. Therefore, the Lagrangian in this case will need to have  $SU(3)$  gauge invariance. This group can be expressed in a singlet and an octet representation. Once again, massless field vectors will appear, but this time there will be eight of them. They are the eight gluons introduced in section 1.4. The Lagrangian for QCD will look like this.

$$\mathcal{L} = i\hbar c \bar{\psi} \gamma^\mu \partial_\mu \psi - mc^2 \bar{\psi} \psi - (q \bar{\psi} \gamma^\mu \lambda \psi) \cdot \mathbf{A}_\mu - \frac{1}{16\pi} \mathbf{F}^{\mu\nu} \cdot \mathbf{F}_{\mu\nu} \quad (6)$$

$\mathbf{A}_\mu$  are the eight gluon fields and the  $\lambda$  are the Gell-Mann matrices.  $\mathbf{F}_{\mu\nu}$  is defined as

$$\mathbf{F}_{\mu\nu} \equiv \partial^\mu \mathbf{A}^\nu - \partial^\nu \mathbf{A}^\mu - \frac{2q}{\hbar c} (\mathbf{A}^\mu \times \mathbf{A}^\nu) \quad (7)$$



In both QCD and QED the gauge fields that appear in the Lagrangian from requiring local gauge invariance are massless, as a result the mass term in the "free" Lagrangian for those fields vanishes. However, the gauge fields that are associated with the Weak Force are not massless; therefore, the mass term will not vanish; it cannot simply be written into the Lagrangian since it will not preserve local gauge invariance.

In 1961, Glashow began to formulate a theory where the electromagnetic and weak forces were not separate entities, but instead two manifestations of the same unified force. In 1967, Weinberg and Salam helped formalize this theory into what is known as Electro-Weak Theory. In this theory, only left handed particles, helicity equal to negative one, couple with the weak gauge fields. The property that couples to the weak force is called weak isospin. Each generation of left-handed leptons form an isotopic doublet, and each right-handed lepton forms its own isotopic singlet. The left- and right-handed leptons transform in the following way:

$$\psi_L \rightarrow e^{i\alpha(x)\cdot\mathbf{T}} e^{i\beta(x)\frac{Y}{2}} \psi_L \quad (8)$$

$$\psi_R \rightarrow e^{i\beta(x)\frac{Y}{2}} \psi_R \quad (9)$$

Both  $\alpha(x)$  and  $\beta(x)$  are arbitrary real functions.  $Y$  is the Weak Hyper charge and is related to the electric charge,  $Q$ , and the third component of isospin,  $I^3$ .  $Q = I^3 + \frac{1}{2}Y$ . These transformations form a group of  $2 \times 2$  matrices which belong to  $SU_L(2)$  and  $1 \times 1$  matrices which belong to  $U_Y(1)$ . This gauge theory has an  $SU_L(2) \times U_Y(1)$  symmetry.

The Lagrangian will contain the following terms for the fermions and gauge fields:

$$\bar{\psi}_L \gamma^\mu (i\hbar c \partial_\mu - \frac{1}{2} g \mathbf{T} \cdot \mathbf{W}_\mu - \frac{1}{2} g' Y B_\mu) \psi_L + \bar{\psi}_R \gamma^\mu (i\hbar c \partial_\mu - \frac{1}{2} g' Y B_\mu) \psi_R - \frac{1}{16\pi} \mathbf{W}^{\mu\nu} \cdot \mathbf{W}_{\mu\nu} - \frac{1}{16\pi} B^{\mu\nu} B_{\mu\nu} \quad (10)$$

Here  $\mathbf{T} = \tau^+ \hat{x} + \tau^- \hat{y} + \tau^3 \hat{z}$ ,  $\tau^\pm = \frac{1}{2}(\tau^1 \pm i\tau^2)$ , and the  $\tau^{1,2,3}$  are the Pauli matrices.  $\mathbf{W}_\mu = W_\mu^+ \hat{x} + W_\mu^- \hat{y} + W_\mu^3 \hat{z}$ .  $W_\mu^\pm$  are the wave functions representing the charged intermediate vector bosons,  $W^\pm$ . This accounts for the two charged gauge bosons in this theory, but there are still two more neutral bosons:  $Z_\mu^0$ , the third massive weak gauge boson, and  $A_\mu$ , the photon. They are related to the third component of  $\mathbf{W}_\mu$  and  $B_\mu$ .

$$A_\mu = B_\mu \cos\theta_w + W_\mu^3 \sin\theta_w \quad (11)$$

$$Z_\mu^0 = -B_\mu \sin\theta_w + W_\mu^3 \cos\theta_w \quad (12)$$

$\theta_w$  is called the weak mixing angle and is a parameter of the theory. Its experimental value is approximately  $30^\circ$ . At this point, the electro-weak Lagrangian has many terms, none of which resemble mass terms similar to the ones in the QED and QCD Lagrangians. It appears that all of the particles involved, including the fermions, are massless.

In order to account for the masses of the weak gauge bosons and the fermions involved in this theory, a scalar field is introduced into the Lagrangian and a technique called “Spontaneous Symmetry Breaking” is employed. This scalar field is also an isotopic doublet. The Lagrangian contains these terms for the new scalar fields:

$$|(i\hbar c\partial_\mu - \frac{1}{2}g\mathbf{T} \cdot \mathbf{W}_\mu - \frac{1}{2}g'Y B_\mu)\phi|^2 + \frac{1}{2}\mu^2\phi^*\phi - \frac{1}{4}\lambda^2(\phi^*\phi)^2 \quad (13)$$

Both  $\lambda$  and  $\mu$  are real constants. The potential associated with this new scalar field has multiple minima, which correspond to multiple ground states. If an expansion is done about one of the minima with a change of variables, the Lagrangian will transform and new terms will arise. One of these terms will be a mass term for this new scalar field, and there will also be a few leftover terms involving a massless scalar boson. These boson are known as Goldstone Bosons and are undesired. However, with a clever choice of a local gauge transformation, the undesired boson will be transformed away. All that will be left is a Lagrangian with mass terms for the appropriate leptons and gauge bosons that were in the theory from the start, and a new massive scalar boson known as the Higgs Boson.

To see how the masses of the  $W^\pm$  and  $Z^0$  arise, simply apply this spontaneous symmetry breaking technique to the first term in Eqn. 13. Two mass terms will arise, one for the  $Z^0$  and one for the  $W^\pm$ s. Using  $\bar{g} = \sqrt{g^2 + g'^2}$ ,  $g/\bar{g} = \cos\theta_w$ , and  $g'/\bar{g} = \sin\theta_w$ , along with the constants in the mass terms, this simple relation between the masses of the  $W^\pm$  and  $Z^0$  arises:

$$\frac{m_W}{m_Z} = \cos\theta_w \quad (14)$$

In addition to the terms in Eqn. 10 and Eqn. 13, the Lagrangian will contain terms involving  $\phi$ , the left handed doublets, and the right handed singlets for the fermions, both for quarks and leptons. Using the same technique as above will give rise to mass terms for all of the fermions.

It is important to note that unlike the leptons, the left handed doublets for the quarks are not simply each generation of quarks. Instead, there is mixing between generations with the weak force. The left handed Doublets are:

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L, \begin{pmatrix} t \\ b' \end{pmatrix}_L$$

where

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

This matrix is called the Cabibbo Kobayashi Maskawa quark mixing matrix. The values in this matrix describe the relative likelihood that a quark will decay into another lighter quark via a weak process. For example,  $|V_{tb}|^2$  is the probability that a top quark will decay into a bottom quark. If the CKM-matrix was diagonal, then there would be no mixing of quarks between generations and weak decays between generations of quarks should not be observed experimentally. However, the CKM-matrix is not diagonal and this is well supported by experiment. The magnitude of the values of the CKM-matrix are:

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.97419 \pm 0.00022 & 0.2257 \pm 0.0010 & 0.00359 \pm 0.00016 \\ 0.2256 \pm 0.0010 & 0.097334 \pm 0.00023 & 0.0415^{+0.0010}_{-0.0011} \\ 0.00874^{+0.00026}_{-0.00037} & 0.0407 \pm 0.0010 & 0.999133^{+0.000044}_{-0.000043} \end{pmatrix}$$

## 1.6 The Top Quark

After the discovery of the bottom quark in 1977, and the measurement of its weak isospin,  $T_3 = -\frac{1}{2}$ , physicists began searching for its bigger brother, the top quark. Until the early 1990s, these searches increased the lower limits of its mass. In April of 1994, the top quark was discovered by CDF in proton-antiproton collisions in the Tevatron. The top quark completed the third generation isospin doublet with  $T_3 = +\frac{1}{2}$ . It has a spin of  $\frac{1}{2}\hbar$  and an electric charge of  $+\frac{2}{3}e$ .

The Tevatron produces top quarks and anti-top quarks by colliding protons and anti-protons at a center of momentum energy  $\sqrt{s} = 1.96\text{GeV}$ . Top quarks are produced by quark-antiquark annihilation or gluon-gluon fusion, with  $q\bar{q}$  annihilation responsible for over 90% of  $t\bar{t}$  events.

After the collision, a gluon is produced that can subsequently decay into a top and anti-top quark pair if the gluon has sufficient energy. As can be seen in the CKM matrix, the top quark almost exclusively decays into a bottom quark and a  $W^+$  boson. Likewise, the anti-top quark's favored mode of decay is into an anti-bottom quark and a  $W^-$  boson. Both the bottom and anti-bottom quarks decay and hadronize in the detector and produce a jet. The  $W^\pm$  bosons have two major decay modes; one which decays hadronically into a  $q\bar{q}$  pair that hadronize into two energetic jets, the other is a leptonic decay that produces a charged lepton and a neutrino.

Given all the different type of decay products,  $t\bar{t}$  events can be classified as being all hadronic, lepton plus jets, and di-lepton events. The following figure displays all of the different decay modes of  $t\bar{t}$  events and their branching ratios where  $q\bar{q}'$  signifies any quark-antiquark pair except for  $t\bar{t}$ .

Decay Mode	Branching Ratio	Decay Channel
$t\bar{t} \rightarrow (q\bar{q}'b)(q\bar{q}'\bar{b})$	36/81	All Hadronic
$t\bar{t} \rightarrow (q\bar{q}'b)(e^-\bar{\nu}_e\bar{b})or(e^+\nu_e b)(q\bar{q}'\bar{b})$	12/81	Lepton + Jets
$t\bar{t} \rightarrow (q\bar{q}'b)(\mu^-\bar{\nu}_\mu\bar{b})or(\mu^+\nu_\mu b)(q\bar{q}'\bar{b})$	12/81	
$t\bar{t} \rightarrow (q\bar{q}'b)(\tau^-\bar{\nu}_\tau\bar{b})or(\tau^+\nu_\tau b)(q\bar{q}'\bar{b})$	12/81	
$t\bar{t} \rightarrow (e^+\nu_e b)(\mu^-\bar{\nu}_\mu\bar{b})or(\mu^+\nu_\mu b)(e^-\bar{\nu}_e\bar{b})$	2/81	Di-Lepton
$t\bar{t} \rightarrow (e^+\nu_e b)(\tau^-\bar{\nu}_\tau\bar{b})or(\tau^+\nu_\tau b)(e^-\bar{\nu}_e\bar{b})$	2/81	
$t\bar{t} \rightarrow (\mu^+\nu_\mu b)(\tau^-\bar{\nu}_\tau\bar{b})or(\tau^+\nu_\tau b)(\mu^-\bar{\nu}_\mu\bar{b})$	2/81	
$t\bar{t} \rightarrow (e^+\nu_e b)(e^-\bar{\nu}_e\bar{b})$	1/81	
$t\bar{t} \rightarrow (\mu^+\nu_\mu b)(\mu^-\bar{\nu}_\mu\bar{b})$	1/81	
$t\bar{t} \rightarrow (\tau^+\nu_\tau b)(\tau^-\bar{\nu}_\tau\bar{b})$	1/81	

All hadronic  $t\bar{t}$  events result in six energetic jets: two from the bottom quarks and four from the decays of the  $W^\pm$  bosons. Given the six jets, and identifying which of them were the decay products of the bottom quarks along with requiring that the masses of the top and anti-top quark be equal, it is possible to kinematically reconstruct the mass of the top quark. This channel has a branching ratio of approximately 44%, the largest of the three. While this channel will produce the largest number of  $t\bar{t}$  events, which makes it attractive statistically, it also has the largest background of the three channels. It is plagued by higher order QCD processes that also produce multiple jets.

In the lepton plus jets channel, only one of the  $W^\pm$  decays hadronically while the other decays leptonically. Like the all hadronic channel, there are six decay product, three jets, one of which

is a b-jet, one charged lepton, and one neutrino which escapes detection. The signature for this channel is therefore a lepton plus three or more jets. Due to the escape of the neutrino, the mass of the top quark can not be kinematically reconstructed, because the problem is not fully constrained. The branching ratio for this channel is also approximately 44%; however, events involving the short lived  $\tau$  lepton are typically not included, which drops the effective branching ratio to 30%. This channel is also contaminated by events involving higher order QCD processes, though the requirement of an energetically charged lepton makes the QCD background a smaller set than the all hadronic channel; however, this channel also includes a new set of backgrounds involving the direct production of  $W^\pm$  bosons.

The di-lepton channel contains events where both of the  $W^\pm$  decayed leptonically. Similar to the lepton plus jets channel, events involving the  $\tau$  lepton are not included which drops the effective branching ratio from approximately 11% to 5%. In addition to having a small branching ratio, this channel has the disadvantage of having two undetectable neutrinos. The signature of the channel is therefore two highly energetic charged leptons and two energetic jets. The major advantage of this channel is that its two charged leptons help to distinguish it from many other type of multi jet events and therefore it has the lowest background of the three channels.

The di-lepton channel is used exclusively in this dissertation. The following figure is a Feynman diagram that illustrates the production and decay of a  $t\bar{t}$  event in the di-lepton channel.

